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Can a Tachyon Emit Light Radiation in All Directions?

G. ALAGAR RAMANUJAM

Department of Physics, NGM College, Pollachi 642001, Tamil Nadu (India)

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Abstract

It is shown here that a critical analysis of the approaches employed by various authors to accommodate tachyons into special relativity leads one to the conclusion that a tachyon can emit light radiation only along its line of motion.

1. *Introduction*

The problem of determining Lorentz-type transformations in four-dimensional space between two inertial frames (hereafter called S and S') with a superluminal relative velocity (v) along their common x axis has received considerable attention in recent years (Recami & Mignani, 1972; Goldoni, 1973; Alagar Ramanujam & Namasivayam, 1973; Alagar Ramanujam, 1974; Mignani & Recami, 1974). The approaches employed in these references, though slightly different from each other in some technical details, are all based on a common philosophy that particles behaving as tachyons with respect to subluminal observers will behave as bradyons with respect to superluminal observers (principle of duality). It is shown here that a critical analysis of these approaches enables one to conclude that a tachyon can emit radiation only along its line of motion. First, we give below the salient features of the different approaches.

2. *The RM and GAN Approaches*

In the approach employed by Recami and Mignani (hereafter called the RM approach), the most important point to be noted is this: the transverse coordinates y and z of a point in space take imaginary values in one frame and real values in another frame. This is really embarassing.[1] An equally

[1] It is true that E. Recami and R. Mignani have time and again explained the sense in which the appearance of i in their formulas is to be viewed. We feel that such explanations are not fully convincing.

embarrassing feature of the approach employed by Goldoni and by Alagar Ramanujam & Namasivayam (hereafter called the GAN approach) is the necessity of introducing a non-Euclidean geometry in a three-dimensional flat space. If U_x, U_y, and U_z are the three components of the velocity of a light photon emitted by a tachyon, then according to the GAN approach, c^2 is not equal to ${U_x}^2 + {U_y}^2 + {U_z}^2$ but is equal to ${U_x}^2 - {U_y}^2 - {U_z}^2$! The transformation laws between the two frames S and S' are as follows:

$$x' = R(x - vt) \tag{2.1}$$

$$t' = R(t - vx/c^2) \tag{2.2}$$

$$R = 1/\sqrt{v^2/c \;\; - 1} = 1/\sqrt{\beta^2 - 1}$$

$$y' \text{ or } z' = iy \text{ or } iz \quad \text{(the RM approach)} \tag{2.3}$$

$$= y \text{ or } z \quad \text{(the GAN approach)} \tag{2.4}$$

$$U'_x = \frac{U_x - v}{1 - vU_x/c^2} \tag{2.5}$$

$$U'_{y,z} = \frac{i\sqrt{\beta^2 - 1}}{(1 - vU_x/c^2)} \quad \text{(RM approach)} \tag{2.6}$$

$$= \frac{U_{y,z}\sqrt{\beta^2 - 1}}{(1 - vU_x/c^2)} \quad \text{(GAN approach)} \tag{2.7}$$

3. *The Radiation from a Tachyon*

Let us now consider a photon emitted by a light source kept at the origin of the frame S' in a direction making an angle θ' with the x' axis in the $x'y'$ plane of the frame S'. Then $U'_x = c \cos\theta'$ and $U'_y = c \sin\theta'$ (we imagine the frames to be situated in vacuum, where the velocity of light is c). Transforming the velocity components to the frame S, for which the source will be a superluminal one (i.e., a source of light moving with a velocity greater than c) we have

$$U_x = \frac{c\cos\theta' + v}{1 + v\cos\theta'/c} \tag{3.1}$$

from which we have

$$U_x = c \text{ or } -c$$

when $\theta' = 0°$ or $180°$, for which $U_y = 0$ and

$$U_x > c$$

when $0° < \theta' < 180°$ and $180° < \theta' < 360°$ for which $U_y \neq 0$. From this it follows that only the photons emitted in the directions $\theta' = 0$ and $\theta' = 180°$ can meet the requirement of the constancy of the velocity of the light in S, i.e. $U_x{}^2 + U_y{}^2 = c^2$, and the photons emitted in all other directions cannot meet the above relativistic requirement, since in such cases U_x itself is greater than c. We interpret this result in the following way: Any event which goes to violate the principle of the constancy of the velocity of light is unphysical and hence is not observable. In the above case, the photons emitted in directions other than $\theta' = 0°$ or $180°$ do not satisfy the requirement of the constancy of the velocity of light in the frame S; and they are therefore unobservable for an observer in S. For the observer in S, only the photons emitted in the directions $\theta' = 0°$ or $180°$ are observable, or in other words, as far as the observer in S is concerned the light source which has a superluminal velocity emits radiation only along its line of motion. (It is true that the above requirement of the constancy of the velocity of light can be met by the photons emitted in *all* directions if one banks upon the imaginary value of U_y and U_z as given in the RM approach or if one takes into account the non-Euclidean geometry as given in the GAN approach. But it must be pointed out that both these concepts are purely a mathematical necessity rather than a logical deduction based on physical facts. So, instead of going after such unphysical concepts, we feel it is better to stick to our own usual notions of space and geometry and face the consequences.) We generalize the above result saying that a superluminal particle can emit radiation only along its line of motion. For example, a superluminal particle moving from a point P to a point Q can emit radiation only along the line joining P and Q. As a consequence of this, it follows that all the events that are observable in both the frames lie along the line of relative motion between the frames. Any event occurring away from this line can be observed only by one of the frames but not by both. Since the question of finding transformation laws arises and has meaning, only when both the frames observe a particular event, the transformations between the frames S and S' become purely a two-dimensional affair, in the (x, ict) plane. (The line of relative motion can be taken as the x axis.) We therefore write

$$x' = R\,(x - vt) \qquad t' = R\,(t - vx/c^2) \tag{3.2}$$

$$y' = y = 0 \quad \text{and} \quad z' = z = 0 \tag{3.3}$$

It must be stressed here that the relation (3.3) is not at all a matter of convenient choice but is rather dictated by the requirement of the constancy of the velocity of light in superluminal frames. It may be noted that in light of the relation (3.3) the RM and GAN approaches are happily relieved of their embarrassing feature that we mentioned in the beginning of this paper, and thereafter they become identical with each other.

4. *The Superluminal Doppler Effect*

From what has been said above it is clear that in the case of a source of light moving with a superluminal velocity, we can have only a longitudinal Doppler effect, but not a transverse one. The treatment given below for the longitudinal Doppler effect is based on certain results obtained by Alagar Ramanujam (1974) in a recent paper where it was shown that the invariance of light velocity, when extended to superluminal frames, gives rise to two distinct cases. They are when

$$u = -c, \quad u' = -c \tag{4.1}$$

and when

$$u = +c, \quad u' = -c \tag{4.2}$$

Accordingly we have, for the first case,

$$p = R(p - E/c^2 v) \tag{4.3}$$

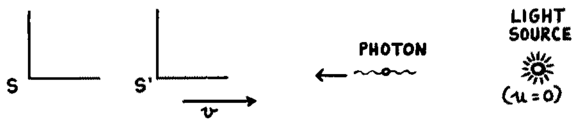
$$E' = R(E - vp) \tag{4.4}$$

and, for the second case,

$$p' = R(p - E/c^2 v) \tag{4.5}$$

$$E' = R(vp - E) \tag{4.6}$$

The first case corresponds to a situation when a light source fixed ($u = 0$) with respect to the observer in S, at a point on the positive side of the common x axis, is observed by the observers in S and S'. This is symbolically given in the following figure:



For the observer in S', the light source will be an approaching one ($U' = -v$). Replacing P', p, E', and E by h/λ', h/λ, $h\nu'$, and $h\nu$ in (4.3) and (4.4), we have

$$h/\lambda' = R(h/\lambda - v/c \cdot h\nu/c) \tag{4.7}$$

and

$$h\nu' = R(h\nu - vh/\lambda) \tag{4.8}$$

After a rearrangement we get

$$\lambda' = \lambda\sqrt{(\beta - 1)/(\beta + 1} \tag{4.9a}$$

$$\nu' = \nu\sqrt{(\beta + 1)/(\beta - 1)} \tag{4.9b}$$

The second case corresponds to a situation when a light source kept at the origin of the frame S is observed by the observer in S'. In this case, for the frame S', the light source will be a receding one. Repeating the above substitutions in Eqs. (4.5) and (4.6), we have

$$\lambda' = -\lambda\sqrt{(\beta + 1)/(\beta - 1)} \tag{4.10}$$

$$\nu' = \nu\sqrt{(\beta - 1)/(\beta + 1)} \tag{4.11}$$

The negative sign appearing in the value of λ' in (4.10) only means that, for the frame S', all the light photons from the source appear to travel in the negative direction of the x' axis.

5. *Conclusion*

To conclude, we would like to recall the inference deduced in this paper, namely that a tachyon can emit radiations only along its line of motion. The validity of this inference clearly hinges on the validity of the principle of the invariance of light velocity for superluminal frames. Now, one can argue as follows: Einstein's postulate regarding the principle of the invariance of light velocity is only an abstraction from and simplification of an actual observation expressed by the negative result of Michelson and Morley's experiments performed in a subluminal frame. How far, then, are we justified in simply extending this principle to superluminal frames? Further, one can follow Eddington's philosophy that an essential difficulty in a theory can always be traced to an epistomological error—to a wrong or too narrow concept—and can argue that the appearance of the imaginary coordinates in one case (the RM approach) and the non-Euclideam geometry in the other case (the GAN approach) may be a clear indication that our extension of the principle of the invariance of the light velocity to superluminal frames may be a wrong or too narrow concept. Though we admit that these arguments do create certain reasonable doubts, we strongly believe that Einstein's axioms of special relativity, which have predicted so many seemingly unbelievable things and which have withstood the test of time for the past seven decades, are essentially true for superluminal frames also.

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